Least median of squares: a suitable objective function for stock assessment models?

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Abstract: Robust fitting methods, intended for data sets possibly contaminated with invalid observations, are gaining increased use in analysis of fishery data. In particular, the method of least median of squares (LMS) has attracted attention. Its hallmark is high statistical resistance, which makes it immune to up to 50% contamination in the data. However, the same property makes it inefficient and can cause faulty fitting of typical fishery data. The LMS fit can be in conflict with important sections of a time series, a problem we illustrate by fitting a biomass dynamic (surplus production) model to simulated and actual fishery data. Additionally, we illustrate that LMS parameter estimates can be highly sensitive to small perturbations in the data. Other robust methods, like the method of least absolute values (LAV), appear less prone to such problems. A key reference on LMS recommends using the method as part of an exploratory procedure to identify outliers, rather than as an objective function for final model fitting. We concur with that recommendation.

Résumé : Les méthodes d'ajustement robustes destinées aux ensembles de données potentiellement contaminées par des observations invalides sont de plus en plus utilisées dans l'analyse des statistiques de pêche. En particulier, la méthode des moindres médianes des erreurs au carré (LMS) a suscité beaucoup d'intérêt. Sa caractéristique principale est sa forte résistance statistique qui lui permet de supporter une contamination représentant jusqu'à 50 % des données. Cependant, cette même caractéristique la rend inefficace et peut occasionner un mauvais ajustement de données du type généralement obtenu dans les pêches. L'ajustement de LMS peut être en désaccord avec de longues sections des séries temporelles; c'est un problème que nous illustrons en ajustant un modèle dynamique de biomasse (de production excédentaire) à des données simulées et réelles de pêche. De plus, nous démontrons que les estimations des paramètres de LMS peuvent être très sensibles à de petites perturbations dans les données. D'autres méthodes robustes, telles que la méthode des moindres valeurs absolues (LAV), semblent moins sujettes à ces problèmes. Un travail essentiel sur les LMS recommande d'ailleurs d'utiliser la méthode comme une des procédures exploratoires pour identifier les données aberrantes, plutôt que comme fonction objective pour l'ajustement final du modèle. C'est là une recommandation que nous entérinons.

[Traduit par la Rédaction]

Introduction

Most fishery models are fit, in whole or in part, by least-squares (LS) estimation. Some models explicitly use LS as the objective function; others use maximum likelihood or conditional maximum likelihood, which in many cases are equivalent to LS. Least squares has an extensive literature, and its estimates have many attractive features; however, a widely recognized drawback of LS is its sensitivity to outlying values (Rousseeuw and Leroy 1987). Thus, in a relatively small data set, typical in fishery work, a single outlying observation that results in a large residual (or a few such observations) can have relatively strong influence (in the ordinary sense) on the resulting estimates. In many cases, one would prefer that the most outlying points not have such great influence, and it is for such cases that robust fitting methods have been devised.

When errors satisfy the Gauss–Markov assumptions of independence and identical $N(0,\sigma^2)$ distribution, LS is optimal in linear regression in the sense of providing minimum variance, unbiased estimates (Casella and Berger 1990). Because of its desirable properties in linear fitting, LS is widely used in nonlinear fitting, as well. However, real data sets often violate the Gauss–Markov assumptions, and that can have substantial effects on the LS parameter estimates. In particular, data used in stock assessments are likely to violate those assumptions because of large measurement and process errors (Hilborn and Walters 1992) that may not be normally distributed, even under transformation. Poorly defined error structure and the widespread existence of outliers in fishery data sets make robust methods attractive for fitting stock assessment models (Chen and Jackson 1995, 2000).

There are two approaches to using robust methods to reduce the impact of outliers (Rousseeuw and Leroy 1987). The first approach identifies outliers based on the residuals from a robust model fit. Once identified, outliers are corrected, removed, or downweighted; then the "good" data are refit using standard LS methods. This approach was illustrated in fishery analyses by Chen and Paloheimo (1994), Restrepo and Powers (1997), and Prager (2002), among others.

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The second approach to using robust methods simply replaces the LS objective function with one less sensitive to outliers, i.e., a robust objective function. That approach was illustrated by Chen and Andrew (1998), who proposed use of least median of squares (LMS) as a "potentially useful addition to methods used to fit production models to abundance index and catch data". That paper has been influential: in a recent working group, we observed analysts using LMS as the objective function for production models, citing Chen and Andrew (1998). We found that alarming, because problems can arise with such uses of LMS. The goals of this paper are to illustrate those problems and to discourage the use of LMS as a replacement for standard methods of model fitting.

LMS

The method of LMS is a well-known robust fitting method. As with any regression, the goal of LMS is to estimate p model parameters, $\boldsymbol{\theta} = (\theta_1,\ldots,\theta_p)$, from the data. A LMS estimate $\hat{\boldsymbol{\theta}}$ is one that minimizes the objective function

$$(1)\qquad \text{median } r_i^2, \qquad i = 1,2,\ldots,n$$

where the residuals, r_i, are the differences between observed and predicted responses.

LMS is devised to have a high breakdown point, usually defined as the smallest percentage of "contaminated" data needed to shift the estimate by an arbitrary amount (Rousseeuw and Leroy 1987). In linear regression, the LMS breakdown point is 50%; the corresponding LS breakdown point is zero (Rousseeuw 1984).

Other robust methods

Ordinary least squares falls under the more general category of M estimators (Rousseeuw and Leroy 1987; Berk 1990). Such estimators are based on minimizing an objective function of the form

$$(2)\qquad \sum_{i=1}^{n} \rho(r_i)$$

where ρ is a symmetric function with a unique minimum at zero (Rousseeuw and Leroy 1987). For example, $\rho(x) = x^2$ defines the LS objective function. Other functions can be used that reduce the influence of outliers and are thus considered more robust than LS. Such functions generally increase at a slower rate than a quadratic, either for all x or for all x greater than some value. The method of LAV (Harter 1985), a robust M estimator used here, is defined by $\rho(x) = |x|$. Other M estimators worth mentioning, but not receiving full attention here, are the Huber, bi-square, and Bell M estimators (Berk 1990). The Huber estimator uses the LS objective function up to some predetermined residual value, and beyond that uses the LAV objective function; the bi-square estimator uses a function that initially increases quickly (like Huber), but is constant after some residual value; the Bell estimator is similar to bi-square, but with a smoother function that approaches a constant as its limit.

LMS is considered to be an S estimator, which minimizes a type of robust M estimate of scale on the residuals (for detailed description of S estimators, see Rousseeuw and Leroy (1987)). Least trimmed squares (LTS), another widely used S estimator, is LS performed after removing a user-defined fraction of the highest squared residuals (Rousseeuw 1984). Both LMS and LTS attempt to minimize some robust measure of scatter in the residuals.

Nonlinear analyses

In general, stock-assessment models are nonlinear. Robust and resistant objective functions, although initially developed (like least squares) for linear models, can also be applied (like least squares) to nonlinear cases. Unfortunately, minimization of the objective function (i.e., parameter estimation) is usually much more difficult in nonlinear analyses, particularly when the objective function is not smooth (Gill et al. 1981). LMS, being an extreme case of a nonsmooth function, poses a particularly difficult challenge in nonlinear optimization (Stromberg 1993). In general, the properties of LMS in nonlinear analyses have not been as thoroughly studied as in linear analyses. In nonlinear analyses, LMS is considered a high-breakdown estimator, but definition of the breakdown point varies and breakdown analysis becomes more complex (Stromberg and Ruppert 1992). Nonetheless, LMS has been found useful for detecting outliers in nonlinear regression (Stromberg 1993).

Robust methods in fishery management

Robust methods have been applied to several problems in fishery science; we mention only some applications here. Chen and Paloheimo (1994) compared the relative merits of LS, LMS, LMS-based reweighted LS, and LAV for estimating mortality rates and the catchability coefficient from catch and effort data. Prager et al. (1995) used robust regression in fitting length-conversion equations for billfish. Stock–recruitment relationships have been analyzed with a variety of robust methods, using both simulated data and real data on several species (Chen and Paloheimo 1995; Wang and Liu 1999). Robust methods have been used in several studies to fit population dynamic models to time-series data. Such analyses have examined both surplus production models without age structure (Chen and Andrew 1998; Chen and Montgomery 1999; Prager 2002) and assessment models of catch at age (Restrepo and Powers 1997).

Trouble with LMS

In this section, we present four examples that demonstrate inherent difficulties possible when using LMS as an objective function. The first two examples are based on simulated data; the last two are based on real data.

Our first example demonstrates instability of LMS in a linear regression problem. However, the potential for instability in LMS is not limited to linear regression. High-breakdown methods like LMS achieve robustness by essentially ignoring outlying data, and such data may be signal, not noise. Thus, the high-breakdown property can lead to poor fits of distinct sections of data, especially when fitting autocorrelated time-series data.

The remaining examples illustrate potential pitfalls when using LMS to fit a logistic (Schaefer 1954, 1957) surplus production model (here chosen to represent a nonlinear fishery model) to simulated and real data sets on catch and relative abundance. The production model formulation, detailed by Prager (1994) and reviewed by Quinn and Deriso (1999),

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postulates that the rate of biomass change follows the differential equation

$$(3) \qquad \frac{dB_t}{dt} = (r - F_t)B_t - \frac{r}{K}B_t^2$$

Here, B_t is the population biomass at time t, F_t is the fishing mortality rate at time t, r is the population's intrinsic growth rate, and K is the carrying capacity. Typically, F is converted from fishing effort rate f_t assuming constant catchability q so that $F_t = qf_t$. Integrating eq. 3 projects biomass over time and further integration (of F_tB_t) projects the corresponding yields Y_t.

Given observations on effort rates and yields, one can estimate model parameters r, K, q, and B_0. Estimates from surplus production models can be used to derive management benchmarks, particularly maximum sustainable yield (MSY) and fishing effort at MSY (f_{MSY}). Under the assumption of logistic population growth, $\widehat{\mathrm{MSY}} = \hat{r}\hat{K}/4$ and $\widehat{f_{MSY}} = \hat{r}/(2\hat{q})$.

The method of parameter estimation is described in detail by Prager (1994) and was implemented here with the computer program ASPIC (Prager 1995). The fitting method is an observation-error estimator conditioned on observed yield. The estimator was originated in this context by Pella (1967) and was termed the "time-series" method by Hilborn and Walters (1992). Objective functions (LS, LAV, LMS) were computed from residuals in the abundance index in logarithmic transform. In nonlinear LMS analysis, it can be extremely difficult to locate the global minimum of the objective function. The software used requires repeated restarts of its algorithm to the same point to accept a solution, even provisionally. For this study, the fitting software was modified to repeatedly refit over a grid of initial guesses, and the best solution so obtained was adopted.

In examining the results from the three objective functions, we compare predicted and observed relative abundances. We also compare the estimates of four quantities of management interest: MSY, f_{MSY}, F in the last estimated year relative to F at MSY($F_{\bullet}/F_{MSY}$), and stock biomass in the last estimated year relative to the stock biomass at MSY ($B_{\bullet}/B_{MSY}$).

Example 1: instability in linear regression

In linear regression, LMS is robust to 50% contamination of the data, providing a high breakdown point. However, this does not mean, as one might expect, that LMS estimates are insensitive to small perturbations in the data. To the contrary, LMS estimates can be quite unstable (Hettmansperger and Sheather 1992). Although they are insensitive to outliers, LMS estimates can jump between solution regimes in response to small changes in the data, as may occur when adding new observations or with shifts in existing observations.

We demonstrate the potential for LMS instability in a widely familiar setting: linear regression. We generated 31 pairs of (x,y) data according to

$$(4a) \qquad x_i = i, \quad i = 0,\ldots,30$$

$$(4b) \qquad y_i = \begin{cases} 0, & i = 0,1,\ldots,14 \\ 1, & i = 15 \\ 2, & i = 16,17,\ldots,30 \end{cases}$$

We then fit the model $Y = \theta_1 + \theta_2 X$ by estimating θ_1 and θ_2 under three different objective functions: LS, LAV, and LMS. On this data set, the three solutions are quite similar (Fig. 1*a*). An interesting characteristic of LMS is that solutions are not always unique, and here, two distinct LMS solutions exist (Fig. 1*a*).

Two small changes in the data set illustrate the instability of LMS. When the centrally located point (x_{15}, y_{15}) is increased from (15, 1) to (15, 2), the LS and LAV lines hardly change, but the LMS line changes markedly (Fig. 1*b*), from $y = -0.21 + 0.08x$ (or $y = -0.29 + 0.08x$) to $y = 2$, a horizontal line! When, instead, the central point is decreased from (15, 1) to (15, 0), the LS and LAV fits again change very little, but the LMS fit jumps to $y = 0$, a second horizontal line (Fig. 1*c*). This example demonstrates that LMS estimates can be much more sensitive to small data changes than LS and LAV estimates (Figs. 1*a*–1*c*).

By design, the LMS objective function essentially ignores almost half the data. Thus, shifts in which points contribute to the objective function can occur abruptly with small changes in a single data value, leading to marked changes in parameter estimates (Fig. 1*d*). In general, instability of LMS estimates can occur whenever the data can be divided into two partial samples with approximately equal residual values, such that small perturbations can cause the LMS estimate to shift from fitting one partial sample to fitting the other (Hettmansperger and Sheather 1992).

Example 2: instability on simulated fishery data

The next example uses a simulated fishery data set to demonstrate the potential instability of LMS in fishery modeling. We generated time series of fishing effort, biomass, and yield, assuming that stock dynamics follow eq. 3 with $r = 1$, $K = 1000$, $q = 1$, and initial biomass $B_0 = 0.5K$. The applied F relative to F_{MSY} first remains constant at a relatively low level, then increases, and then decreases sharply, as under a sudden, strong management regime (Fig. 2*a*). The resulting trajectory of biomass declines in response to the increased F, and then rebounds (Fig. 2*b*). The simulation provided a noise-free time series of yield, to which we added simulated lognormal observation errors

$$(5) \qquad \tilde{Y}_t = Y_t \exp(v_t), \quad v_t \sim N(0,\sigma^2)$$

We simulated observed yields $\tilde{Y}_t$ using $\sigma = 0.1$, and thus generated a 19-year catch–effort data set. Using the LS, LAV, and LMS objective functions, we then fit the surplus production model in three ways: (*i*) to the first 15 years of simulated data, (*ii*) to the first 17 years of simulated data, and (*iii*) to the entire 19 years of simulated data. This is analogous to repetition of a stock assessment when additional data become available.

On the 15-year time series, the LS and LAV fits are very similar (Fig. 3*a*), and produce similar estimates of management benchmarks. However, the LMS fit is strikingly different, reproducing eight of the data points quite well and seemingly ignoring the other seven (Fig. 3*a*). The LMS estimate of MSY is comparable to those of LS and LAV, but LMS estimates of the remaining management quantities differ considerably (Table 1). With two years of data added to the time series, the LS and LAV fits change little, whereas the

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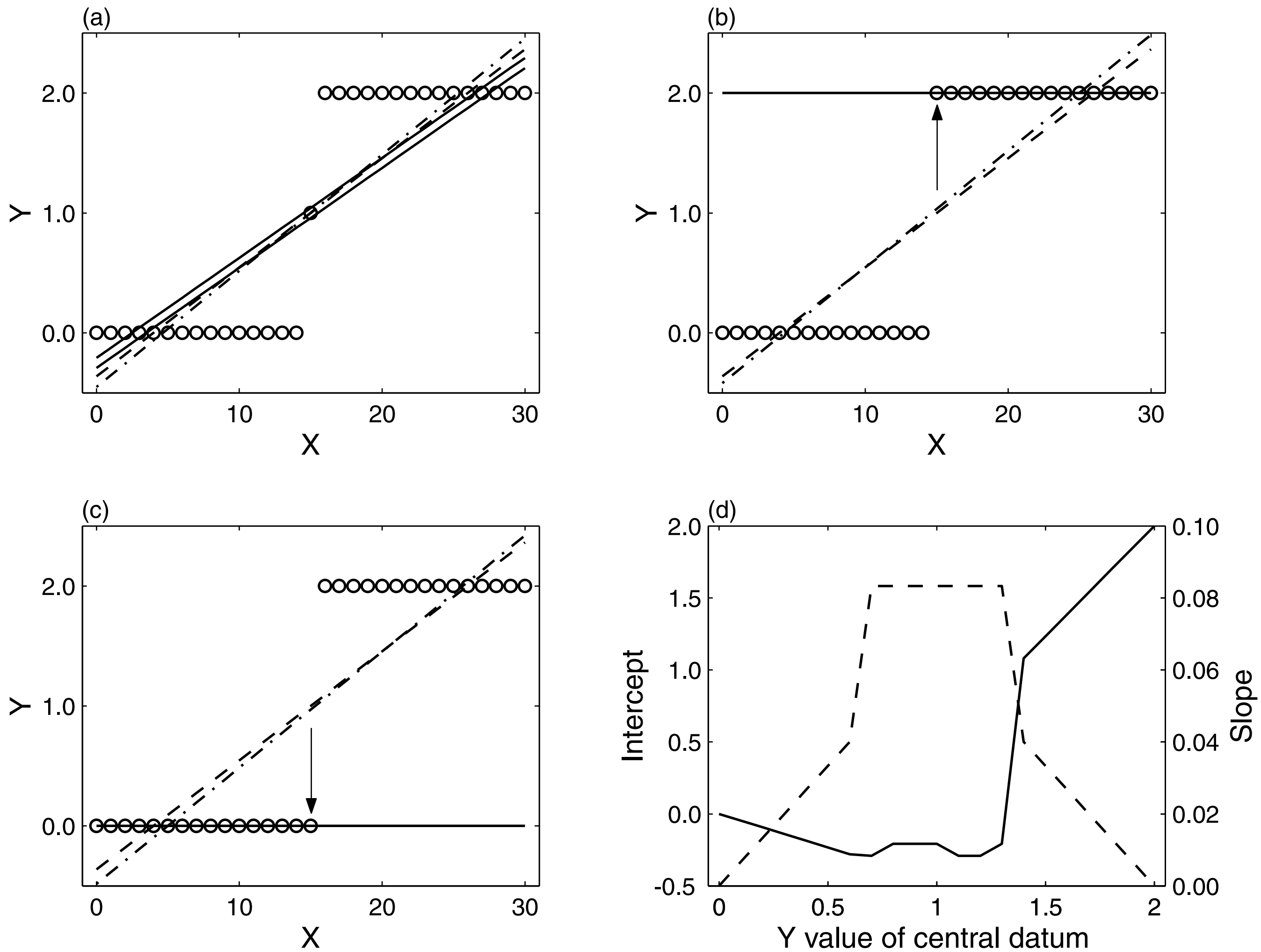
Fig. 1. Examples of linear regression ($Y = \theta_1 + \theta_2 X$) using three different objective functions: least squares (· –), least absolute values (– –), and least median of squares (LMS) (—). LMS can be nonunique and in (*a*) gives two solutions. Data (○) are (*a*) original data (see text), (*b*) central datum shifted up, (*c*) central datum shifted down. (*d*) Sensitivity of LMS estimates, intercept $\hat{\theta}_1$ (—) and slope $\hat{\theta}_2$ (– –), to changes in the *Y* value of only the central datum; remaining data as in Figs. 1*a*–1*c*.



LMS fit jumps to a new solution regime (Fig. 3*b*). This jump is reflected in the estimated management quantities (Table 1). With an additional two years added to form a 19-year time series, the LS and LAV fits still remain largely unchanged, but the LMS fit jumps yet again (Fig. 3*c*). As before, shifts in the LMS-estimated management quantities accompany the change in fit (Table 1). In this example, each time the simulated stock assessment is repeated as additional data become available, estimates from the LMS objective function provide a very different picture of stock dynamics and management implications.

Examples using actual fishery data

The preceding examples used artificial data constructed to illustrate troubles with LMS. We now demonstrate that similar fitting problems can occur on real fishery data sets. As did Chen and Andrew (1998), we fit logistic surplus production models to time series of fishery data. Our examples use data on Atlantic menhaden (*Brevoortia tyrannus*) and barramundi (*Lates calcarifer*). The following analyses are used for illustration and are not intended to define status of these stocks. Although data were obtained from reputable sources, they may include preliminary or unpublished values, and no attempt has been made to verify or update the data sets, nor to compare results to those of other models that might be applicable.

Example 3: Atlantic menhaden

Data on Atlantic menhaden are those used in Vaughan et al. (2002), and consist of total landings in weight and estimates of *F* (on a weight basis), derived from virtual population analysis, for age 2^+ fish. Those derived estimates of *F* were used instead of recorded data on fishing effort because of the density-dependent catchability demonstrated in this species (Schaaf and Huntsman 1972).

When the menhaden data are fit by LS, the fitted line of relative abundance attempts to capture the high relative abundances near the start of the time series (Fig. 4*a*). Because those relative abundances are somewhat inconsistent with simple production model dynamics (Vaughan et al.

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Fig. 2. Simulated data used to illustrate fitting problems with least median of squares: (*a*) trajectory of relative fishing mortality rate; (*b*) resulting trajectory of relative biomass.

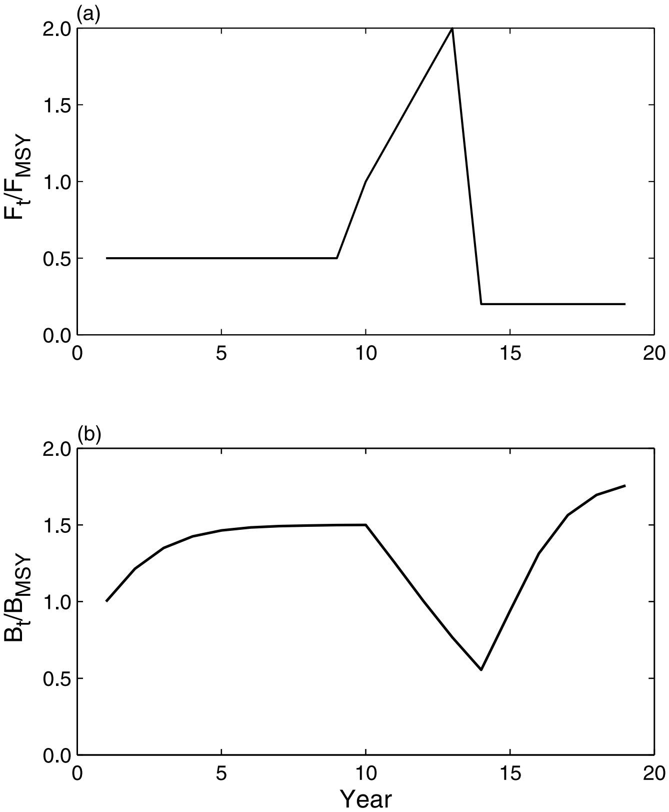
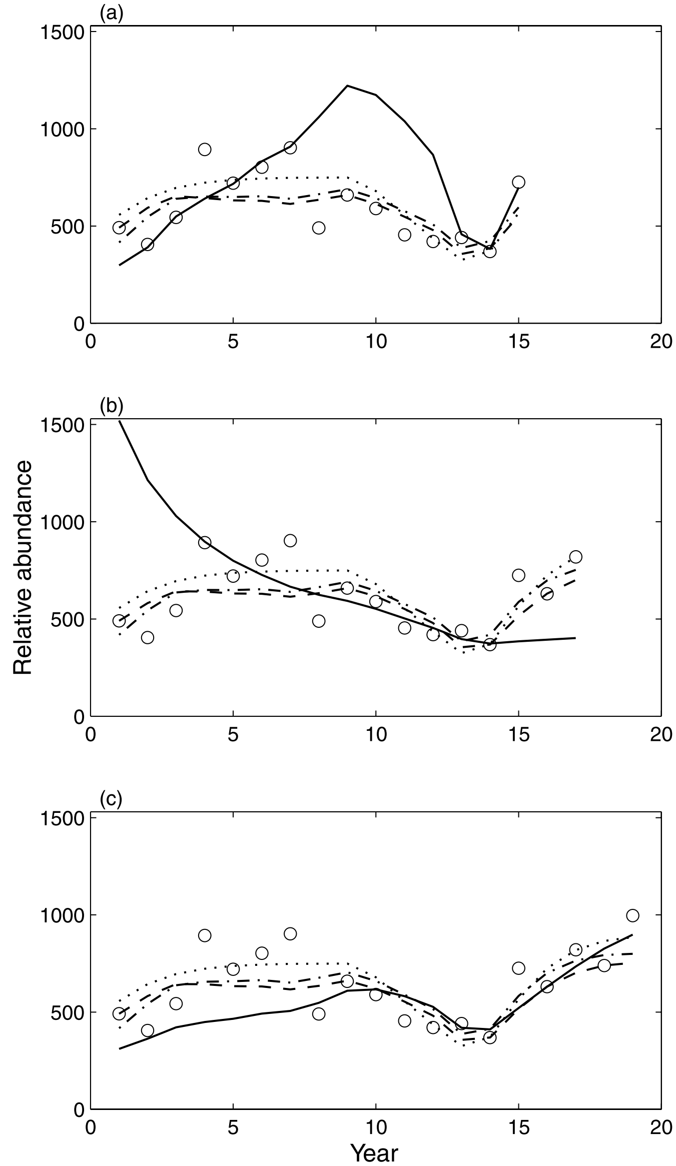


Fig. 3. Production model fits to simulated abundance index data (○) using three objective functions: least squares (· –), least absolute values (– –), and least median of squares (—). "True" underlying simulated values also shown (···). (*a*) Fits based on the first 15 years of data, (*b*) fits based on the first 17 years, and (*c*) fits based on the full data set (19 years).



2002), the fit cannot capture the stock dynamics faithfully. The LAV fit is similar to the LS fit in that the estimated dynamics approximate the observed data (Fig. 4*b*). However, the LMS fit, which aims only at the lowest median residual, essentially ignores the early high abundances, and in those years it predicts much lower abundances than were observed (Fig. 4*c*). The period of high abundance in the 1950s is well documented in the menhaden literature (e.g., Schaaf and Huntsman 1972; Ahrenholz et al. 1987; Vaughan et al. 2002) and is quite unlikely to be an error; thus, the LMS fit conflicts with an important observed phenomenon.

Estimates of management-related quantities from LS and LAV are similar (Table 2). The estimate of sustainable yield (MSY) from LMS is similar to estimates from the other objective functions, but LMS estimates of other management quantities are quite different. The LMS fit describes a stock that supplies the same MSY at higher F_{MSY}, which by necessity would be applied to a smaller B_{MSY} (Table 2).

Example 4: barramundi

Data on barramundi, 1973–1989, were extracted from table 4.1 of King (1995). Data are missing for 1976; our fitting method accommodates the missing value of relative abundance, and we assumed that the 1976 catch was equal to the mean of the 1975 and 1977 catches.

Fits from LS and LAV display similar patterns, differing mostly in the first year (Figs. 5*a*, 5*b*). The LMS fit is less convincing, and resembles the fit to the Atlantic menhaden data in that the initial period of high abundance is not fit at all (Fig. 5*c*). Given the difference in fit, one might expect to see large differences in management estimates from the different objective functions. However, that does not occur in this case (Table 2). It appears that stock dynamics in the latter portion of the data set are sufficiently consistent with the earlier portion that failure to fit the early portion (as in the LMS fit of Fig. 5*c*) does not seriously affect the results.

Discussion

The goal of this study was to illustrate potential problems with use of LMS as an objective function, particularly when fitting time-series data. (We did not attempt to compare

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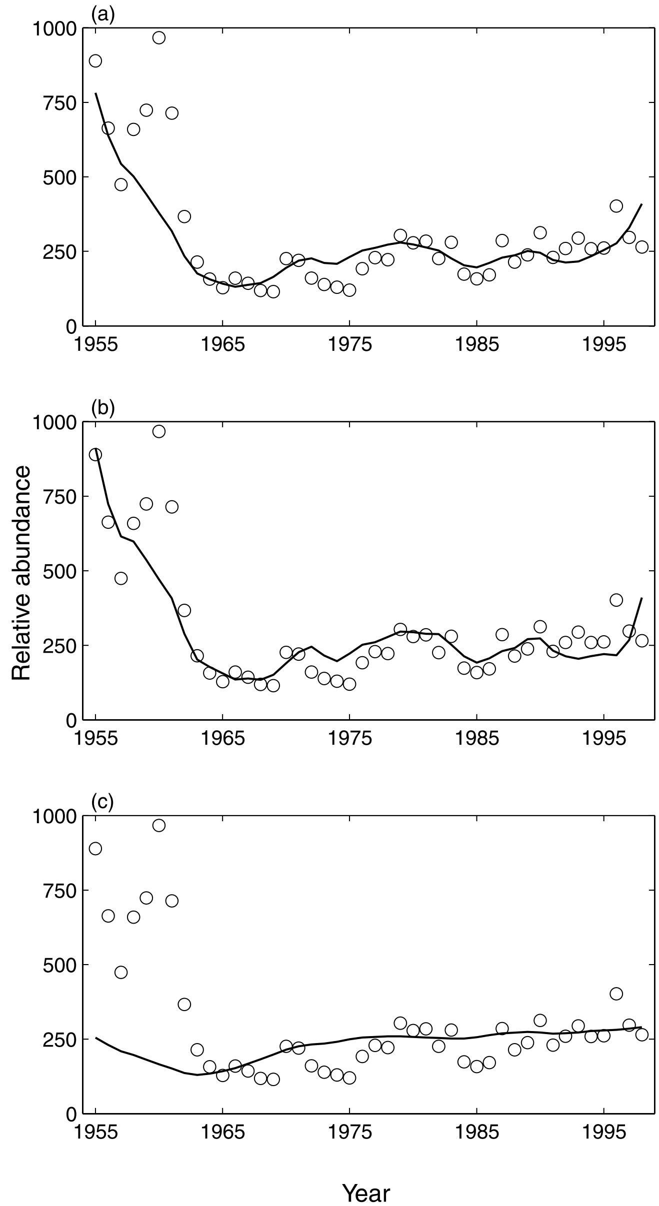
Table 1. Comparison of estimated quantities from application of logistic surplus production model to simulated data under three objective functions and with three sample sizes *N*.

		Estimates			
Objective function	*N*	MSY	f_{MSY}	$F.\!/F_{\mathrm{MSY}}$	$B.\!/B_{\mathrm{MSY}}$
True values	15	250	0.50	0.20	0.94
True values	17	250	0.50	0.20	1.56
True values	19	250	0.50	0.20	1.76
LS	15	271	0.64	0.19	1.54
LS	17	268	0.62	0.17	1.78
LS	19	261	0.59	0.21	1.80
LAV	15	266	0.65	0.20	1.50
LAV	17	251	0.59	0.20	1.71
LAV	19	250	0.59	0.23	1.77
LMS	15	251	0.29	0.36	1.00
LMS	17	125	0.42	0.49	1.36
LMS	19	244	0.40	0.28	1.52

Note: "True" underlying simulated values also shown. Quantities estimated: MSY, maximum sustainable yield; f_{MSY}, fishing effort rate at MSY; $F.\!/F_{\mathrm{MSY}}$, ratio of final-year fishing mortality rate to that at MSY; $B.\!/B_{\mathrm{MSY}}$, ratio of final-year population biomass to that at which MSY can be obtained. LS, nonlinear least squares; LAV, least absolute values; LMS, least median of squares.

Fig. 4. Production model fits (—) to observed abundance index data (○) on Atlantic menhaden (*Brevoortia tyrannus*) using three objective functions: (*a*) least squares, (*b*) least absolute values, and (*c*) least median of squares.



LMS to other objective functions as to bias or variance of estimates: to do so would require extensive simulation studies.) We have encountered problems with LMS in our own work, and given the increasing interest in robust methods in general and LMS in particular, we believe that it is important for fishery biologists to be aware of potential shortcomings. A recurring pattern is that the LMS objective function can ignore portions of the data in searching for the lowest median residual. Such portions can be distinct sections of a time series that carry signals of biological importance. This is implicit in the definition of LMS, but it may not be appropriate for analysis of fishery time series.

At first glance, the problem of ignoring contiguous observations may seem of no importance when fitting non-time-series data (e.g., length–weight data). However, the same issue can arise whenever data cluster into distinct groups that can be described by different parameter values. When more than 50% of the data fall into one such group, the LMS solution can reflect only that group. When the data form two distinct groups, there will be two candidate LMS solutions, potentially with very different parameter estimates, and the choice between them will depend on the relative sample sizes and dispersions in each group. Small data changes (e.g., adding an observation) can move the solution from describing one group to the other, as illustrated in our examples and discussed by Hettmansperger and Sheather (1992) and Ellis (1998). Any data set, even a noisy one, in which roughly half the data define one set of parameter estimates and the other half define a second set, would be subject to instability as data are added or small changes made to central values.

The desired property of LMS is its resistance to outliers. However, resistance comes at a cost: loss of efficiency (i.e., slower convergence of the estimator to its true value with increasing sample size). Stromberg (1993) summarizes research on this topic, including a proof that the asymptotic efficiency of LMS in linear regression is zero. That finding is supplemented by simulations showing that, in linear regression, LMS is quite low in efficiency compared to LS on finite samples with normal errors (Rousseeuw and Leroy 1987). The tradeoff between resistance and efficiency is evident in other robust methods, and in general, higher breakdown is accompanied by lower efficiency (Berk 1990). In general, such issues have been poorly studied in nonlinear applications, in part because of the large universe of nonlinear models.

A practical problem with the use of LMS is the difficulty of finding the minimum of a LMS objective function, partic-

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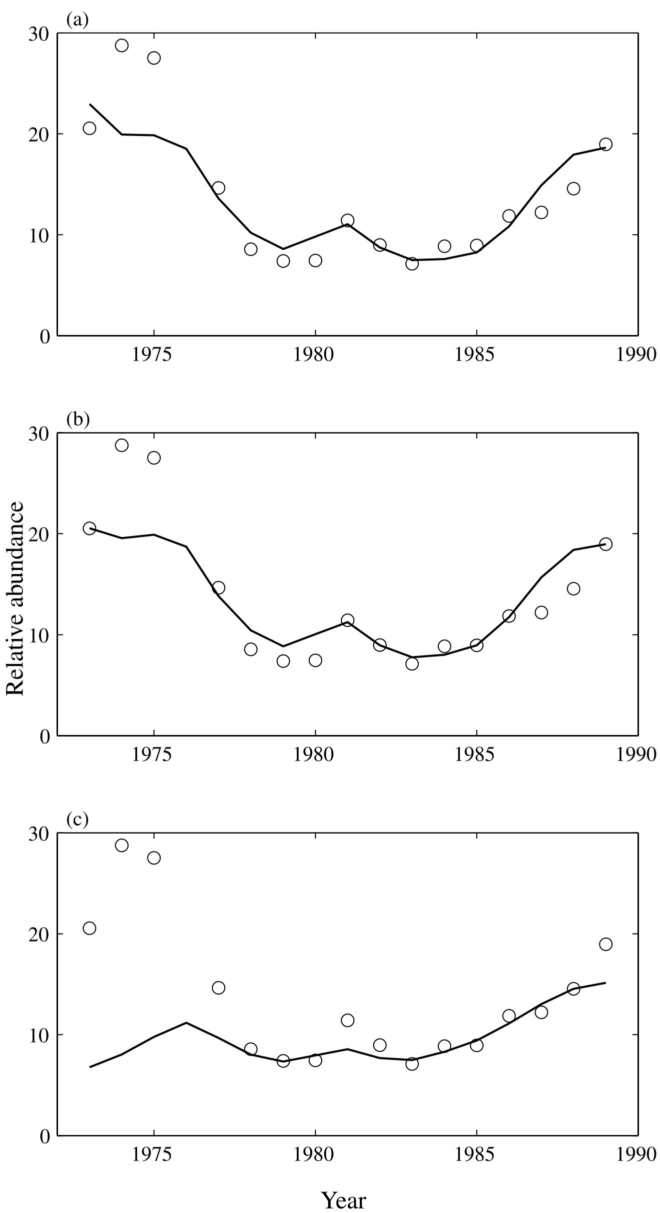
Table 2. Comparison of estimated quantities from application of logistic surplus production model to fishery data sets under three objective functions.

Species	Objective function	MSY	f_{MSY}	$F./F_{MSY}$	$B./B_{MSY}$
Atlantic menhaden	LS	461	0.95	0.63	0.94
Atlantic menhaden	LAV	530	0.86	0.69	0.83
Atlantic menhaden	LMS	402	2.04	0.42	1.48
Barramundi	LS	750	59.1	0.54	1.46
Barramundi	LAV	747	58.0	0.54	1.47
Barramundi	LMS	758	72.3	0.54	1.45

Note: Quantities estimated: MSY, maximum sustainable yield; f_{MSY}, fishing effort rate at MSY; $F./F_{MSY}$, ratio of final-year fishing mortality rate to that at MSY; $B./B_{MSY}$, ratio of final-year population biomass to that at which MSY can be obtained. These examples are not intended to define the status of any stock. LS, nonlinear least squares; LAV, least absolute values; LMS, least median of squares.

Fig. 5. Production model fits (–) to observed abundance index data (○) on barramundi (*Lates calcarifer*) using three objective functions: (*a*) least squares, (*b*) least absolute values, and (*c*) least median of squares.



ularly on nonlinear data (as here). This difficulty, which is recognized in the statistical literature (Stromberg 1993), has at least two components. First, the LMS objective function is not differentiable, nor is it continuous, a situation that precludes the use of gradient optimization methods and makes locating a global minimum much more difficult for other methods (Gill et al. 1981). Second, the LMS solution to a particular problem can be nonunique (as in our first example), and even when the solution is formally unique, it may be nonunique in practical terms. By that, we mean that many LMS minima may exist, all with extremely small residuals. In production modeling, we have found data sets with numerous local minima in which the median residual was less than 0.01% of the actual abundance-index value. As such minima can correspond to markedly different sets of parameter values, we consider such solutions nonunique in practical terms. Differences in the LMS objective function among such local minima can be small enough to result from numerical issues such as roundoff error (in data entry or computation) or choice of convergence criteria.

Because of the troubles associated with using LMS for fitting data, we recommend that it be used in detection of outliers, rather than as a substitute for the usual least-squares objective function in fishery population models. Outlier identification is the use advised by Rousseeuw and Leroy (1987), a standard statistical treatise on robust and resistant linear methods. It has been advocated elsewhere in the fishery literature (Chen and Paloheimo 1994; Chen et al. 1994; Chen and Jackson 2000), though not always adhered to in practice (Chen and Montgomery 1999). By placing LMS in a diagnostic role, the analyst will be in a position to examine LMS results and determine how they might be useful in each particular case.

LMS offers two main advantages in outlier detection. First, outliers cannot always be detected simply by plotting the data, particularly in multivariate data. In such situations, LMS provides an objective method for diagnosis. Second, outliers may exist in independent variables as well as (or instead of) in dependent variables. Such outliers, called "leverage points", can have relatively small residuals, making outlier identification based on these residuals misleading. At least in linear analyses, LMS is robust to outliers in both dependent and independent variables (Rousseeuw and Leroy 1987).

Once identified, outliers in fishery data can be corrected, removed, downweighted, or perhaps included based on background information related to the investigation (Chen and Jackson 1995; Restrepo and Powers 1997; Prager 2002).

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Then, the possibly modified data set can be fit by some more efficient method, like LS (Rousseeuw and Leroy 1987). An alternative might be to fit all the data using a robust method that does not suffer the problems outlined for LMS. For this, we suggest that investigators consider the method of LAV. That method has the advantages of using all the data and yet being more robust to outliers than LS. In addition, it is relatively simple to implement and similar enough to LS that it can be explained easily. A potential drawback of LAV is that it is not robust to leverage points. However, in the context of fitting time-series data, where the independent variable is the year of observation, it seems unlikely that leverage points would exist.

In conclusion, naive use of the method of LMS can be dangerous, particularly when fitting time series or other autocorrelated data. This danger applies to other robust methods in proportion to their statistical resistance. The method of LAV, though robust, does not share LMS's high resistance, and it may thus be more desirable. In all cases, it seems preferable that outlier downweighting and subsequent fitting be done through a process that involves human thought, and not be relegated to an automatic procedure using a robust or resistant method. The unquestioned value of such methods is in detecting outliers. The decision to change or discount the data is best left to the subject-matter expert.

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